



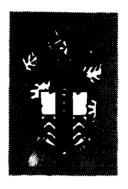
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### RSRE MEMORANDUM No. 3532

### ROYAL SIGNALS & RADAR ESTABLISHMENT

PREDICTIVE DISPLAY DESIGN FOR A TWO-AXIS CONTROL TASK

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PROCUREMENT EXECUTIVE, MINISTRY OF DEFENCE, RSRE MALVERN, WORCS.

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AUTHOR:

P J Goillau

DATE:

August 1982

### SUMMARY

A simulation study has been carried out using a generalised two-axis control and guidance task to investigate predictive display design parameters. Predictive displays are found to bring about a substantial improvement in man-machine system performance over unaided manual control. Recommendations are made regarding the choice of prediction model and prediction span as a function of controlled system dynamics and input uncertainty.

This study highlights the need to augment human anticipatory skills in complex control tasks, and illustrates a methodology for optimising predictive display design which could be extended to serospace and missile guidance applications.

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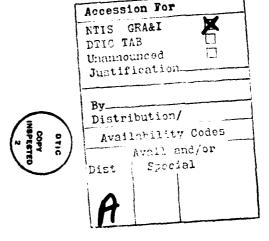
### P J Goillau

### LIST OF CONTENTS

- 1. Introduction
- 2. Predictive display design parameters.
- 3. Detailed experimental method.
- 4. Results.
- 5. Discussion.
- 6. Conclusions.
- 7. Acknowledgements.
- 8. References.

Figures 1 - 10

Tables 1 - 3



### 1. INTRODUCTION

Piloting an aircraft or firing a missile are examples of complex control and guidance tasks found in modern warfare. The human operator in such tasks exercises effective control by constructing, through training and experience, an 'internal model' of the process dynamics. Using this model he can anticipate the trajectory resulting from his control actions based on the current displayed position. In general, the more complex the process the longer it will take the operator to build his model, and the more flawed that model will be. Laboratory studies confirm that human predictive skills are inadequate for all but the most elementary processes (Sheridan and Rouse 1971, Rouse 1973, Van Heusden 1977).

The problem is particularly acute in flight systems and missile guidance, where both training and operational errors are costly. Simulators are now extensively used in both these areas, so that the pilot or gunner may acquire as effective a model as he is capable of within the given training time constraints. However, the fact that operational hit rates in manually-guided missile systems tend to be low suggests that some form of automatic aiding would be beneficial. However, total automatic control of either a flight or a missile system is clearly undesirable if mission flexibility and manual override are to be retained. For aircraft, a recent advance has been to feed into a mathematical simulation of the aircraft dynamics the pilot's control actions and such environmental factors as can be measured, and to generate in real-time a predicted future trajectory for the aircraft. Such a display is termed a predictor or predictive display.

The predictive display concept was first derived by Kelley in 1958, based on Ziebolz and Paynter's (1954) theory of two-time scale computing. In the last 20 years the value of predictors has been well documented, mostly in military and vehicular simulations as far removed as aircraft landing, VTOL and helicopter hovering, air-traffic control, lunar rovers, spacecraft guidance, docking ocean-going vessels, submarine depth control and remote control guidance systems. This work is reviewed in Warner (1969), Smith and Kennedy (1975) and Goillau (1978). Towill (1980) further advocates the use of microelectronic digital predictor systems for advanced aerospace applications.

Predictor displays have been shown to be reliably superior to 'quickened' displays (McLane and Wolf, 1966). Common findings are that learning times are reduced, often to the point where novice operators are able to control relatively complex systems with essentially no training whatsoever. In addition, human operator performance is substantially improved in terminal control tasks and in controlling non-linear systems or linear systems with pure time delays. Predictor-aided manual control can approach optimal control with respect to a particular performance criterion, as the operator can plan optional courses of action to increase the likelihood of mission success. Lastly, the information processing requirements on the human operator can be reduced, particularly in multi-dimensional control tasks.

The technique is equally applicable to missile guidance, where a computer-generated missile flight path could be superimposed onto an optical or thermal sight, as in Figure 1. This approach transforms a conventional two-dimensional (x, y) representation of the three-dimensional (x, y, t) problem, where the time axis must be inferred by the operator, into an alternative representation where the time component is made explicit by the predictor trace. A further advantage is that the predictor trace, once aligned, could be used to automatically home the missile to target proximity in the closing stages of an engagement via a 'dead man's handle' arrangement, even after the operator or his sight had been rendered inoperative by enemy action.

Whilst the efficacy of predictive techniques has been well documented, there has been little work on the optimum design of predictor display systems to match a particular application. The present study reviews the more important parameters affecting predictor display design, and demonstrates how these may be optimised experimentally for a given system.

### 2. PREDICTIVE DISPLAY DESIGN PARAMETERS

The main parameters inherent in any closed-loop manual control task incorporating a predictor display are:

- 1. System dynamics gain and control order (effective number of integrations).
- 2. Prediction model fidelity the accuracy with which the prediction model represents the controlled system's behaviour.
- 3. Prediction span (extrapolation interval) the real-time period over which predicted system response is displayed. This is frequently the same as the prediction time the real-time interval over which the future system response is computed by the prediction model.
- 4. Input uncertaintly a measure of the accuracy of the updating information fed to the prediction model, including the effect of external disturbances such as crosswinds.
- 5. Repetition (refresh) rate the number of successive predictions displayed to the operator per unit of time, often the frequency at which the prediction model is itself updated.
- 6. Mode of control refers to whether the predictive display is arranged in on-line, off-line, or supervisory configuration.

Factors peculiar to manual control systems not listed above mainly involve display formats, and the problem of which system variables need to be displayed

to the operator. The answers to such questions are system specific. No general quantitative studies of predictor characteristics have been conducted, however, and as the following discussion of predictor characteristics and their interactions implies there is clearly much to be learned in this area.

### 2.1 SYSTEM DYNAMICS

Several workers have addressed experimentally the question of system gain and control order. Warner (1969) reports that in his terminal control task operator performance was independent of system parameters over the ranges investigated, ie from high gain (short response times) to low gain (long response times). However, the more important system-orientated performance measures did show a dependence upon predictor system parameters. It appears that the sensitivity of the performance measure(s) to control action timing is an important factor. Bernotat and Widlok (1966) found that their extrapolative predictor display improved the quality of control over a no predictor condition for all values of system gain investigated, but the improvement was most pronounced at high gains. In absolute terms, error scores using the predictor reached a minimum level for medium gain and rose at low and high gains.

As far as control orders are concerned, Bernotat and Widlok (1966) report that in their stabilisation problem the greater the number of process integrations, the larger was the benefit obtained from the predictor. Bernotat (1972) notes that, as control order increased from two to three integrations, errors rose considerably as did the amount of control effort required of the operator to achieve that level of performance.

Rouse (1970) summarises the situation when he suggests that predictive aids may be of maximum benefit in tasks of medium difficulty: they are probably unnecessary for easy tasks, and in very difficult tasks the operator is so overloaded that he will ignore the information. Rouse's experimental evidence supports this suggestion.

### 2.2 PREDICTION MODEL FIDELITY

A 'perfect' predictor instrument is one which completely predicts the future state of a controlled process, by displaying to the operator one or more future states in addition to the present system state. It is a hypothetical concept. As previously noted, three classes of prediction fidelity have been put forward by Bernotat and Widlok (1966):

CLASS I PREDICTION uses a mathematical power series to extrapolate repetitively from the current value of the controlled system and its derivatives. System time-history is ignored in favour of present movement. As the prediction process is independent of system characteristics, Class I prediction cannot be used to predict accurately far into the future. Its necessarily short prediction spans are mainly applicable to stabilisation and guidance problems, the absolute value of the prediction span being dependent on the dynamics of the system and its external disturbances. This technique has been widely used by the West German school (Bernotat, 1972; Dey, 1972). Bernotat and Widlok (1966) note that although the extrapolations are not as accurate as (for example) Class II prediction, because man extrapolates very coarsely the method is more accurate than anything the human can manage and in most cases will suffice to provide some lightening of the load. Some degree of model inaccuracy can also be tolerated due to the human operator's adaptability.

Extrapolation according to this method is a problem of approximation, the function being approximated by a power series. A Taylor series expansion is typically used, of the general form:

$$y (t + \theta) = y(t) + \sum_{n=1}^{N} \frac{\theta^{n}}{n!} \cdot \frac{d^{n}}{dt^{n}} y(t)$$

where N is the number of derivative terms in the extrapolation. Dey (1972) notes that the order of extrapolation should be one less than the order of the controlled system. For example, a controlled process having three integrations in the forward path would require a second order extrapolation. It is a property of the Taylor series expansion that the deviations from the true path will increase with increasing prediction time. Accuracy on the other hand depends on how many terms of the series are used, and this is limited by the computational resources. It is also influenced by the level of noise contamination, which will be amplified as N, the number of derivative terms, is increased.

CLASS II PREDICTION differs in that it assumes the controlled system's transfer function or response characteristics are known and can be included in the prediction model. Usually an analog model of the process (though there is no reason why this should not be accomplished digitally -Towill (1980)) is run in 'fast-time' alongside the real-time system to be controlled. The fast-time model is fed with exactly the same control inputs as the real-time process, and so extrapolates the predicted path of the system from its present state. Because of its greater accuracy Class II predictive displays permit prediction further into the future than Class I. However, the two-time scale modelling technique does not achieve perfect extrapolation since it does not include factors such as crosswinds external to the system; hence unlimited length prediction spans are not possible. Its applications include long term stabilisation and guidance problems, and this approach has been developed and widely used by the American school, notably Kelley (1958, 1960a, b, 1962, 1968, 1972) and his colleagues. Most of the documented applications centre around Class II instruments. It is worth pointing out that the Class II approach can be thought of as providing the best estimate of all the terms in a Class I Taylor series expansion; since the Class II predictor is exact all but for external disturbances.

CLASS III PREDICTION approaches the hypothetical perfect predictor in that important external disturbances which are to some degree predictable are included in the fast-time model. Obviously the incorporation of all possible system disturbances (in a space mission for instance) tends to stretch computing facilities to their limit, and therein lies the principal drawback of this method. Class III predictive displays can, however, be used to extrapolate far into the future, and their long and accurate prediction spans provide a useful navigational feature.

In practical terms, Class II prediction will usually suffice for most operational accuracy requirements since rapid updating of predictions will tend to offset external disturbances. However, if Class I can be successfully used a substantial saving in required computational power will result.

### 2.3 PREDICTION SPAN

There is conflicting evidence as to how far ahead prediction should extend in the predictive display. Kelley (1960a) in an early predictive

display study found that whilst approximate prediction models could be of some assistance, useful prediction spans decreased with decreasing model fidelity and learning times for effective control were increased. Bernotat and Widlok (1966), however, report the opposite. As the order of their extrapolation model was reduced, useful prediction times increased by a few seconds.

Subjects in a submarine control task (Kelley, 1960b), when permitted to adjust prediction span, elected to reduce it as vehicle speed was increased. Kelley (1962) thus recommends that slow, sluggish systems such as submarines are best served by a long prediction span (25 to 30 seconds), whereas quick-changing, high-frequency systems such as helicopters require a shorter span (5 seconds). Dey and Johannsen (1969) on the other hand suggest that the faster the control task, the longer the prediction time span should be. Dey (1971) also found optimum prediction time to increase as the controlled process increased from a second to a third order system. The latter authors were concerned with extrapolative predictive displays for VTOL aircraft hovering.

In the limiting case, extremely short spans provide insufficient information and control instability ensues. An unnecessarily long span is, by definition, unnecessary and may even act as system noise. It is not known whether such noise distracts the operator and degrades his performance, or merely acts as superfluous information, the research findings being inconclusive. Rouse (1970) for example found that a 40 second span yielded worse overall performance than a 20 second span in an aircraft guidance task, as subjects wasted time correcting distant errors that would never arrive. Williams (1969) using an aircraft predicted pitch display reported that performance remained the same with spans of 3.5 to 6.75 seconds, but deteriorated for spans of less than 3.5 seconds.

Besco (1964) in a simulated spacecraft attitude control task evaluated prediction spans from 10 to 30 seconds but found no significant difference on performance, perhaps due to inaccuracies in the prediction model used. McLane and Wolf (1967) investigating predictor displays for submarine course and depth control also reported no significant difference between prediction spans of 20, 30 and 40 seconds, though the 40 second span did result in larger overshoots. There was also some evidence that had a more stringent tip-of-predicted-path-in-circle tracking task been employed, r.m.s. error would have risen with lengthening prediction spans.

In a study of a simulated jet aircraft landing (Kennedy et al. 1975) control performance not only increased sharply as spans increased from 5 to 20 seconds, but there was an indication that much higher performance would occur with even longer spans. Yet in a follow-up study using 'experienced' subjects from the first experiment, the authors found no difference between spans of 10, 20 and 30 seconds (Smith and Kennedy, 1975). Perhaps a wide range of spans may be equally effective for experienced operators. Smith and Kennedy note that their experience in the Dunlap Labs, where Kelley also carried out much of his work, indicates that operators make use of the first or central segment of a predictor trace rather than its end-point. This procedure effectively minimises the time to reach the desired trajectory. In cases where time is not critical, however, there is probably no advantage in using any particular segment of the trace, again suggesting that a broad range of prediction spans may be equally facilitating.

Practical considerations usually require the selection of one prediction span, unless the operator is given the freedom to adjust the

prediction span for himself. Different systems will undoubtedly need different prediction spans, related to the 'responsiveness' of the system and to the magnitude and frequency of unpredictable disturbances. Bernotat (1972) in this context comments that the proper choice of prediction time can improve performance by as much as 70%. Kelley (1960b) has noted that for some tasks span should be in terms of distance rather than time. Rouse (1970), Dey (1971) and Bernotat (1972) have all found optimum prediction spans/times in laboratory simulation studies.

### 2.4 INPUT UNCERTAINTY

This refers to the accuracy of the information input to the prediction model, and is not the same as prediction model fidelity. Input uncertainty is caused by normal variability in system operation or by external disturbances to the controlled system (eg. cross winds affecting aircraft flight, transmission noise on signal lines). The net effect of these variations is, however, similar to prediction model inaccuracies in that they both serve to reduce the credibility of the predicted information displayed to the operator. If the uncertainty cannot be incorporated into the predictive trace, for example because its form is entirely unknown, then there will be a discrepancy between actual and predicted paths which can only serve to mislead the operator. In this case the useful prediction span may have to be reduced. If however the nature of the uncertainty can be measured or forecast and incorporated into the predicted trace then the display has the addition of a diagnostic feature. One suggested approach is to display multiple predicted paths corresponding to the mean predicted path with extreme ranges to either side. There has been little quantitative research in this area (see Tainsh, 1977).

### 2.5 REPETITION (REFRESH) RATE

Repetition (refresh) rate of the display is the number of successive predictions displayed to the operator per unit of time. In theory for fast-time models it is determined by the prediction model time scale, the prediction span, and a negligible amount of time spent in updating or resetting the model. In practice the maximum repetition rate is determined by the limits of the computer one is using, and may be quite low (in the order of seconds). With low repetition rates the information conveyed by the predictor trace becomes more out of date as the cycle proceeds, and the predictive display itself acts as a sampled data system. Low repetition rates may also cause display flicker and associated visual fatigue problems for the operator as well as control difficulties. In general the required repetition rate increases as system response becomes more rapid.

The frequency at which the predictor model is updated with fresh information is often identical to the repetition rate (any faster would be pointless), in which case its effects are synonymous. When updating frequency is lower than the repetition rate the first prediction after updating will be the most accurate and each successive prediction will decrease in accuracy until the model is again updated. One solution to this dilemma is to update the prediction model artificially by extrapolating past sampled outputs of the system over the updating period; another would be to let the predictor sample its own predictions and so update itself. For most applications repetition rate and frequency of updating are the same and are predetermined by the computer system. In any case their effect is likely to be slight. McCoy and Frost (1966) report that reducing the updating frequency of their predictor from continuous updating down to once every 50 seconds apparently made no difference to

performance. The practical significance is that prediction model inaccuracies can sometimes be offset by a high frequency of updating.

### 2.6 MODE OF CONTROL

Two principal modes of control may be distinguished, depending on the philosophy behind the predictive display in use and the application for which it has been designed. These are on-line control and off-line control, of which category exploratory control and supervisory control (monitoring) are special cases. Figure 2 illustrates the main differences.

In <u>on-line control</u> the input to the prediction model is identical to the control input to the system itself, so the operator sees a predicted path based on the assumption that he does not alter his control input. Any control change is immediately reflected on the predictive display. This mode of control is particularly suited to situations where an 'ideal' path or trajectory can be formulated, eg aircraft landing. The pilot, via a continuous series of trial-and-error control actions in fast-time with real-time effects, is able to reduce the difference between actual and desired trajectories until his plane is on the runway glidepath.

In off-line control a hypothetical input is fed to the prediction model based on the assumption that the operator's control action will change during the predicted interval. The hypothetical input may take the form of sampled present control inputs or a complex pre-programmed sequence of control actions yielding a display of several different responses, the so-called 'multiple path prediction'. Exploratory control is a special case of off-line control. It differs from on-line control in that the operator's control actions are not input to the plant until he decides that the results of his choice of action, as reflected on the predictor display, constitute the optimum solution. In effect his control is directly coupled to the predictor display but indirectly coupled to the process, via an appropriate switch or sample-and-hold circuit. The selected control action may be the operator's most recent manipulation, or one that has previously been placed in 'storage' (Kelley, 1968). A variation of this technique is the case where the operator adjusts a hypothetical control program, building up a sequence of control actions, and only then does he command the actual controller to assume the form (in real-time, naturally) of the hypothetical program. Kelley et al (1973) have termed this flexible approach 'automanual control'.

It is evident that all forms of off-line control presuppose the luxury of sufficient time to explore the potential effects of alternative control actions. If appreciable searching is required before the best performance is reached, on-line control may be inadvisable as it will lead to substantially higher fuel consumption (McCoy and Frost, 1966). Warner (1969) has shown that exploratory control is marginally better than on-line control (though not statistically different), so long as the required decision times are not short. Where control decisions are required immediately, however, on-line control is generally to be preferred. The additional control errors and use of fuel and resources attributable to an on-line mode of control are usually negligible when compared to the consequences of a long decision time.

The fourth mode of control, supervisory control, can be though of as a further special case of off-line control and refers to those situations where the primary mode of control is automatic. The human functions in a system monitor capacity and may override the automatic system in cases of

emergency ("reversion" in missile systems), system failure, or for maintenance. Strictly, the entire control system is on-line, while the automatic and manual components are on-line and off-line respectively. The prediction model in this case also contains a fast-time model of the automatic controller. Two variations of supervisory control are possible, differing in the degree of 'pureness' of the off-line component. In cases where the automatic control system is malfunctioning and manual back-up is essential, the operator may have little or no time to explore the utility of various control inputs. In this extreme he will be functioning in an on-line mode. In cases where automatic control mal-function occurs, but time is non-critical, or where the automatic system is functioning correctly but unanticipated events demand manual override, then the operator may be functioning predominantly in an off-line mode.

The simulated dual-axis control task described in this report is an example of on-line prediction.

### 3. DETAILED EXPERIMENTAL METHOD USED IN THIS STUDY

A laboratory task was sought which could be generalised to a variety of control and guidance situations and which would facilitate the detailed investigation of a number of predictor display design features. A two-axis compensatory control task was chosen, the object being to keep the system output position in each of two axes simultaneously within certain limits. To facilitate later analysis, each axis was displayed and controlled separately.

### 3.1 THE SIMULATION

The simulation consisted of two independent unstable channels having identical third-order dynamics (Figure 3). Each channel comprised three integrations in series with a digital potentiometer, and was driven by an error signal (E) derived from the subject's control input (Vin) minus a disturbance level (d). The disturbance level varied as a 'random walk' in its magnitude, duration and direction, but on average changed once every 10 seconds or so. The output (Vout) from each channel controlled the position of a pointer set against the calibrated scale of a vertical meter. The value of the digital potentiometer determined the system gain in each axis.

### 3.2 THE TASK

The experimental subject's task was to anticipate the path of two pointers moving against vertical scales calibrated 1 - 100 and by his control actions to maintain the pointers within close proximity of the 50 mark, keeping both pointers simultaneously between 45 and 55 on the scales. The display arrangement is shown in Figure 4, and the control unit in Figure 5. A seconds elapsed time clock was provided on the display, together with an indication of how many seconds the pointers had been simultaneously within limits.

The pointers could not be viewed simultaneously but were instead selected by pressing a button above the appropriate slider on the control unit. To assist control, the option of a predictor trace was provided extending to the right of both pointers. Prediction was based either on an approximately accurate Taylor series extrapolation model (Tay) using the three most recent data points, or on a 'Perfect predictor' model (PPM) based on the simulation itself run in fast-time. The Taylor series extrapolation model took the form:

 $y(t + \theta) = y(t) + \dot{y}(t)\theta + \dot{y}(t)\theta^{2}/2!$ 

where  $(t + \theta)$  is the predicted value of y at  $\theta$  seconds ahead of current time t.  $\mathring{y}(t)$  and  $\mathring{y}(t)$  are respectively first and second order time-derivative terms.

The determination of an appropriate prediction model is largely an engineering problem, and the two models used here were chosen as representing different extremes of computational power requirements. Smoothing problems encountered with the Taylor series approach were overcome by generating derivative terms directly from the simulation, rather than from successive values of the output. In a practical application where one would be forced to use successive values of the output as the basis for calculating derivative terms, output smoothing could be derived by established techniques (moving averages, digital filters).

Initial conditions for the simulation were keyed in by the experimenter from a teletype at the beginning of each trial. Adjustable variables were pointer limits, system gain (effectively system speed of response, achieved by adjusting the potentiometer value), level and timing of the random disturbances, type of prediction model used, prediction span (length of predictor trace from 0 up to 30 seconds), and trial length in seconds. At the end of each trial the total time within limits score was output on the display as feedback to the subject.

### 3.3 EXPERIMENTAL DESIGN

Factors examined in this study were system gain, the level of random disturbances (uncertainty), the prediction model (Taylor series or Perfect Predictor) and the prediction span. The design used was a hybrid between a full repeated measures and a full factorial design, with repeated measures on some of the factors (Winer, 1971; Chapter 7). This type of design has the advantages of tight experimental control and an economic use of subjects.

Each subject underwent three levels of 'uncertainty'/disturbance level  $(\pm~0^{\circ},~\pm~10^{\circ},~\pm~20^{\circ})$ , four levels of prediction span (0, 5, 15, and 30 seconds) and both types of prediction model (Taylor series and Perfect Predictor). The presentation order of the 21 trials was randomised to overcome sequence effects, whilst a thorough training schedule ensured practice effects were minimal.

A total of 15 subjects were used. These were randomly assigned to three independent groups of 5. Each group undertook the experimental trials as described above for one of three levels of gain, corresponding to slow, medium and fast system responsiveness and achieved by adjusting the potentiometer.

The design may be represented as follows:

		NP	Tay	PPM	Tay	PPM	Tay	PPM
		0	5	5	15	15	30	30
	Low Uncertainty	G1	G1	G1	G1	G1	G1	G1
LOW	Medium Uncertainty	G1	G1	G1	G1	G1	G1	G1
	High Uncertainty	G1	G1	Gl	G1	G1	G1	G1
	Low Uncertainty	G2	G2	G2	G2	G2	G2	C2
MEDIUM GAIN	Medium Uncertainty	G2	G2	G2	G <b>2</b>	G2	G2	G2
	High Uncertainty	G2	G2	G2	G2	G2	G2	G2
	Low Uncertainty	G3	G3	G3	G3	G3	G3	G3
HIGH GAIN	Medium Uncertainty	G3	G3	G3	G3	G3	G3	G3
	High Uncertainty	G3	G3	G3	G3	G3	G3	G3

where G1, G2, G3 represent independent groups of five subjects who underwent all Uncertainty, Prediction model, and Prediction span conditions in a randomised order.

### 3.4 PROCEDURE

The experiments were carried out on a PDP-12 computer. On arrival, subjects were given a written set of instructions and the nature of the task was demonstrated. Six training trials were then carried out under a medium level of uncertainty (± 10° disturbance level). A standard training order was used: No Predictor (NP), Perfect Predictor Model (PPM), Taylor series extrapolation model (Tay), PPM, Tay, NP. An abbreviated results printout was obtained for each training trial.

The experimental trials then followed, their order being randomised. Each trial lasted for 5 minutes with a break of 4 minutes between trials. During this time subjects completed a short questionnaire giving their comments on the last trial. A rest period was given midway through the experiment. After all the trials had been run, subjects' overall impressions were noted.

### 3.5 SUBJECTS

The subjects used in this study were undergraduate students. All had some mathematical background.

### 3.6 DATA COLLECTION

An automatic data capture program logged every system input made by the subject, together with system states such as pointer positions, into store at 1 second intervals. At the end of each trial an abbreviated printout of results could be obtained showing time within limits for each pointer and both pointers simultaneously, time spent looking at each pointer, integrated absolute error scores for each channel plus histograms of control actions and pointer positions. In addition the option of a fuller printout giving control positions at 1 second intervals, disturbance levels and last predicted value displayed at 10 second intervals, and a

breakdown of channel switchings could be selected. A comprehensive analysis of each trial was thus possible, which could be matched to subjects' comments.

### 4. RESULTS AND STATISTICS

Time in seconds during which one or both pointers had been outside the prescribed error limits was used as the main performance measure. (In practice, integrated absolute error scores - the total pointer deviations from the 50 scale marker - were found to give similar results, and so are not reported here.)

Group averages of the performance measure in the different experimental conditions have been plotted in Figures 6 - 9. Figure 6 shows the grand averages of the Taylor series extrapolation model and Perfect predictor model for the four prediction spans (each point on the graph is the average of three gains, three levels of uncertainty and five subjects). Figures 7 - 9 expand the basic information of Figure 6 to include the effects of different levels of uncertainty and system gain, separate graphs being drawn for Low, Medium and High gain. Much could also be learnt about individual subjects' control strategies from scrutiny of the control histograms for each trial, from subjective comments and from the completed questionnaires.

Inspection of the time outside limits data showed it to be severely positively skewed. As is appropriate with severely skewed time data of this kind, the within-cell variances were first stabilized before ANOVA analysis by performing a logarithmic transform on the raw error scores, of the form:

$$x'_{ijk} = \log_{10} (x_{ijk} + 1)$$

Ine addition of 1 to each  $x_{ijk}$  term served to prevent the occurrence of  $\log_{10}(0)$ .

The transformed data were analysed in several different ways. A preliminary analysis (Table 1) was used to test for broad differences between the No Predictor, Taylor series extrapolation model (30 seconds prediction span) and Perfect Predictor model (also 30 seconds prediction span) conditions. More detailed analyses were also performed on the full set of Taylor series data (Table 2) and on the complete set of Perfect Predictor data (Table 3). The ANOVA model used was appropriate to multi-factor designs of this type containing some repeated measures, and was followed by tests of simple effects where a significant interaction term had been obtained. Conservative 'F' ratios were employed throughout (Winer, 1971).

### 5. DISCUSSION

### 5.1 TIME OUTSIDE LIMITS DATA

Considering first the preliminary ANOVA (Table 1) which excluded prediction span by comparing scores from the No Predictor condition with scores from the Taylor series and Perfect predictor models having the full 30 seconds prediction span, it can be seen that all the main effects (gain, uncertainty, prediction model) were highly statistically significant, with the complication of considerable interactions.

In general, time outside limits error scores were found to increase with faster system response and with increasing levels of uncertainty. The

significant interaction term (gain x uncertainty) suggests that uncertainty had a differential effect depending on the system responsiveness. The third main effect - that of a NP vs Taylor series vs Perfect Predictor models - was highly significant, coupled to a strong interaction with uncertainty (the Taylor series model was peculiarly immune to variations in uncertainty), and a lesser interaction with system gain.

A major finding is that there was virtually no difference between the two prediction model scores in the Low gain condition (Figure 7). Inspection of the original data shows that near perfect within-limits performance was achieved using the Taylor series extrapolation model as well as with the Perfect predictor trace. This seems to reinforce Kelley's (1960a) and Bernotat's (1972) earlier findings of the effectiveness of simple prediction models. Figure 6 demonstrates that in overall terms, however, the Perfect Predictor was clearly superior to the Taylor series model, especially with longer prediction spans: though either prediction model was preferable to No Predictor at all. It is also evidence from Figure 6 that minimum error scores were achieved with the full 30 second prediction span for the Taylor series model, but with a prediction span of only 15 seconds for the Perfect Predictor trace.

In order to study the interactions with prediction spans in more detail, two separate analyses were performed on the complete data - one analysis of the Taylor series scores and a separate analysis of the Perfect predictor model scores.

### 5.2 INTERPRETATION OF TAYLOR SERIES DATA

The analysis of Table 2 indicates that for the Taylor series prediction model a strong effect due to system gain (0.1% significance) was found, and a somewhat lesser effect (1% significance) due to prediction span. A slight interaction between these two variables was also present. No effect was discovered due to uncertainty, and it is one of the important features of using a Taylor series prediction model that no significant worsening in performance can be expected as the level of input disturbance rises. (The slow response time of such a model may well have served to act as a filter to input noise.)

Because the gain x prediction span interaction was significant, tests on simple main effects were called for rather than further direct testing of the main effects. Results of such tests are given in Table 2a. Examining the interaction effect in more detail suggests that system gain had an increasingly significant effect when any form of Taylor series predictor trace, however short, was introduced. Conversely, the effect of different prediction spans was most marked for slow system response, longer spans resulting in lower error scores, but its effect lessened as the gain was increased. No significant difference between different spans was found in the High gain condition.

Clearly the choice of prediction span using this type of extrapolation model will depend on the gain of the system concerned. For systems with slow or moderate response times the maxim "the longer the better" is valid. For system with very fast response times a slight reduction in prediction span may be advisable on practical grounds.

### 5.3 INTERPRETATION OF PERFECT PREDICTOR DATA

The analysis shown in Table 3 demonstrates that all the main effects

(gain, uncertainty, prediction span) achieved a high degree of statistical significance, in addition to a strong gain x uncertainty interaction term (significant 17), and a lesser uncertainty x prediction span term (significant 57). It is evident that the Perfect predictor model reacted somewhat differently to changes in the experimental conditions than did its Taylor series counterpart. In both cases performance deteriorated as system response speed increased, (Figures 7 - 9) but in contrast to the Taylor series data the Perfect predictor was also adversely affected by increasing the level of uncertainty. This effect was somewhat dependent on the prediction span in use, a more marked deterioration in performance occurring for longer prediction spans. This point will be further discussed below.

Because the two interaction terms achieved significance, tests on simple main effects were again called for rather than further direct testing of the main effects. Findings from the analyses are summarised in Table 3 a, b. Considering first the gain x uncertainty interaction, this point is perhaps of rather academic interest as the analysis is in terms of means obtained by averaging over scores from the four prediction span conditions. It is not discussed further.

Considering the uncertainty x prediction span interaction, test for simple main effects showed no difference due to uncertainty for short prediction spans (0 and 5 seconds), but a highly significant effect (significan 0.1%) for prediction spans of 15 and 30 seconds. Figures 7 -9 represent this information pictorially. In terms of prediction spans, though the effect due to different spans was highly significant for all levels of uncertainty, it was most pronounced at lower uncertainty levels. Inspection of Figures 7 - 9 indicates that an optimum prediction span existed for the Perfect predictor at higher levels of uncertainty. This point had not been revealed by the analysis so far, and so it was decided to carry out trend tests on each gain x uncertainty combination to explore the issue further. The test thought to be most appropriate was Page's L non-parametric test on trends, as this test is more powerful than the omnibus F-test or the equivalent Friedman test (Boersma et al., 1964). The predicted order amongst prediction spans tested was in accord with the cell averages for the Perfect predictor data shown in Figures 7 - 9 and the significance levels obtained are given below:

	LOW UNCERTAINTY		MEDIUM UNCERTAINTY			HIGH UNCERTAINTY						
1 011 CATN	0	5	15	30	0	5	15	30	0	5	15	30
LOW GAIN	pre		ed tr 3. 1%	end		Sig	3. 5%			Sig	3. 1%	
MEDIUM GAIN	0	5	15	30	0	5	15	30	0	5	15	30
redion Gain		Sig	3.0.1%	:		Sig	3. 5%			Sig	3. 1%	
	0	5	15	30	0	5	15	30	0	5	15	30
HIGH GAIN	-	Sia	0.17	•		Sia	0.17	,	_	Sia	0.17	<del></del>

Test for trends amongst prediction spans for Perfect predictor model data It can be seen that the optimum prediction span, ie that giving the lowest error scores, decreased as uncertainty increased. This effect was most noticeable for the High gain (fast response) condition (Figure 9) when the optimum span decreased from approximately 23 seconds to 15 seconds, then to approximately 10 seconds as the level of input disturbance rose. It would seem that operators cannot use the full extent of the Perfect predictor trace due to its long-distance predictive information being rendered inaccurate by input uncertainty. This confirms Rouse's (1970) findings.

Given these interactions, it is clear to see why previous workers have come up with conflicting findings concerning optimum prediction spans. Contrasting the Perfect predictor scores with the Taylor series extrapolation model; only in the High gain condition was there any indication that a reduction in usable span to approximately 15 seconds occurred, but as Figure 9 suggests this effect was nowhere near as significant (Page's L significant at 5%) as for the corresponding Perfect predictor condition (Page's L significant at 0.1%).

### 5.4 CONTROL HISTOGRAMS

Inspection of the control histograms (Figure 10 gives examples) for each trial shows that distinct patterns of control were present for the two prediction models, though of course variations did occur across subjects. Typically, control without any form of predictor was characterised by use of the extreme limits of control in 'bang-bang' fashion. With the introduction of a 5 second Taylor series trace control was still characterised by long periods spent at the extremes, but there was an additional distribution at the centre of the range corresponding to finer control adjustments. This central distribution typically spread towards the extremes as pediction span was extended to 30 seconds.

In the case of the Perfect predictor model, control was characterised by a much smoother gaussian-type distribution. Though for short prediction spans some time was spent at the extreme limits of the controls, this component disappeared as prediction span was increased beyond 5 seconds, and control then consisted of very fine adjustments around the centre of the range. In other words, the fast response of the Perfect predictor model made immediately obvious the effect of a control action and resulted in a smoother pattern of control, whereas the Taylor series rate-of-change predictor required time for a control change to affect it and so control, even if smooth at the start of a trial, frequently ended up at the extremes as the system became progressively more unstable.

The control histograms provided a useful extra indication of performance. In the Low gain condition, for example, although error scroes were much the same for the two prediction models, scrutiny of the control histograms suggests that with the Taylor series model this was achieved by considerably greater control effort. The Perfect predictor gave much smoother control than did the Taylor series equivalent, though it must be stressed that either predictor was preferable to none at all.

The effect of introducing input disturbances to the system was to spread the distribution of control actions towards the extremes, and a similar effect was found when increasing system gain.

### 5.5 DISPLAY SWITCHING

Analysis of the display switching data also revealed some interesting variations in strategy. Display switching was most regular when the

pointers were within the prescribed limits and under control. Switching rates increased as control of the pointer(s) was progressively lost: \*his was most likely to occur for short prediction spans and high gains. In addition, the bias in the time spent looking at the left-or-right-hand meter shifted during the course of a trial, more time being spent attending to the pointer most out of control. In the stable equilibrium state, a switching rate of once every few seconds was typical.

### 5.6 SUBJECTIVE COMMENTS AND QUESTIONNAIRE ANALYSIS

Analysis of subjective comments from the task tended to confirm the impressions gained from the objective analyses. Subjects all stated that they preferred using a predictor trace to control without a predictor, and this was reflected as the "with predictor" trials being rated as easier to control. Opinions were divided for preference between the Taylor series model and the Perfect predictor model. Seven of the fifteen subjects preferred the Perfect predictor trace in that it was a lot more accurate and gave immediate feedback of the consequences of control changes. Five subjects preferred the Taylor series trace because its slow rate-of-change response was easier to follow and gave more time for them to respond. Three subjects failed to detect any difference between the two prediction models. In all, the Perfect predictor model was rated as being more useful than its Taylor series counterpart for a given prediction span. Subjects also rated their control actions as being considerably smoother using the Perfect predictor trace, particularly with longer prediction spans.

On the question of prediction spans, subjects were equally divided in their preference for the longest possible prediction span (30 seconds) or a shorter span (eg 15 seconds). The 5 second span was universally disliked though thought just possible to control with. On their ratings of the predictor's usefulness subjects rated the 30 second span as being most useful in the low gain condition but the 15 second span as being most useful in the faster responding Medium and High gain conditions. Most subjects failed to detect any variations in the level of input uncertainty, though several commented that the pointer appeared to disobey the controls or to move about of its own accord in some of the trials. It was thought more difficult to control these trials (High uncertainty), particularly in the High gain condition using long prediction spans based on the Perfect predictor model, as the variations due to uncertainty in the middle-to-end part of the trace were found misleading. It is interesting to note that with one exception all subjects reported using the end segment of the trace for control - this finding is clearly at odds with reports from the Dunlap labs mentioned earlier. Smith and Kennedy (1975) had noted that their subjects used the first or central segment of a trace in order to effectively minimise the time to reach their desired trajectory. Clearly subjects make full use of the trace they are given.

It is apparent from analysis of the strategies reported by the subjects that anticipation of the pointers' movements played a vital part in control, especially for the No Predictor condition and to a lesser degree in the Taylor series condition. Use of a perfectly accurate prediction model effectively eliminated the need to anticipate the pointers' trajectory. In the No Predictor conditions, subjects followed the strategy of moving the controls to their extreme to compensate as soon as any perceived movement of the pointer was detected. With experience some subjects tried to anticipate the pointers' point of turn and to gradually reduce their control input beforehand. Again, with experience of the

system dynamics (probably gained from the 'with predictor' conditions) some subjects restricted their range of control actions so as not to use the extreme positions, and made their actions consciously smoother.

Subjects varied widely in their ability to verbalise their control processes. One subject reported controlling on the theory that the system dynamics were analogous to simple harmonic motion, another evolved a yo-yo model. It is clear from this admittedly anecdotal evidence that subjects anticipations were based on some crude form of internal model of the process, through which predictions could be made in the absence of a computer-provided prediction. Using the Taylor series predictor subjects frequently used the slope of the trace as the main criterion for the amount of compensation which they applied. Some anticipation was still required, however, and the problem became one of keeping the predictor trace horizontal within the prescribed limits and with the pointer stationary. In the Perfect predictor model conditions, the problem was further simplified and became one of watching the end of the trace and compensating to keep it within limits and as near to the 50 mark as possible. A slightly different policy was adopted if a pointer drifted outside the limits - the object then became to get that pointer back within limits as quickly as possible, if possible keeping the end point of the trace between the limits as the pointer approached them.

### 6. CONCLUSIONS

- 1. A comprehensive study has been carried out using a generalised two-axis control and guidance task to determine how variations in predictor display parameters and task characteristics affect man-machine system performance measures.
- 2. Predictive displays were found to bring about an improvement over unaided manual control in time-on-target scores in all the experimental conditions.
- 3. System speed of response determined the sophistication of prediction model required. For systems with a slow speed of response, there was little to choose between a highly accurate and a relatively unsophisticated prediction model, given that adequate performance with the latter was achieved at the expense of greater control effort. For systems having moderate to fast response times, the more sophisticated prediction model was justified.
- 4. Recommendations can be made regarding the choice of an appropriate prediction span for simple and sophisticated prediction models under various levels of system gain and input disturbance or uncertainty. With simple prediction models, which seemed relatively immune to uncertainty, prediction span was affected by system gain alone. For systems with low to medium gains, the maxim "the longer the better" was appropriate. At high gains a reduction in usable prediction span was advisable. With a hypothetical Perfect prediction model, the optimum prediction span reduced with the combined effect of increasing uncertainty level and increasing system gain.
- 5. Conflicting results of previous workers are explained in terms of the differing gains, levels of uncertainty and prediction models of the systems investigated.
- 6. Reported strategies from subjects in the present study suggest that the formation of a crude form of internal predictive model is an important part of unaided control.
- 7. Coupled with the findings of previous workers, there is an argument for including predictor aiding in future complex manual control applications, such

as flight systems and missile guidance. It is recommended that a simulation approach similar to that described in this memorandum be adopted in order to optimise predictive display parameters for the User's specific application.

### **ACKNOWLEDGEMENT**

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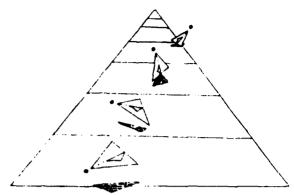
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Spacecraft predictor display shows position and attitude of a tumbling vehicle with respect to a desired command path

Figure la: Predictor display for aerospace applications (after Kelley)

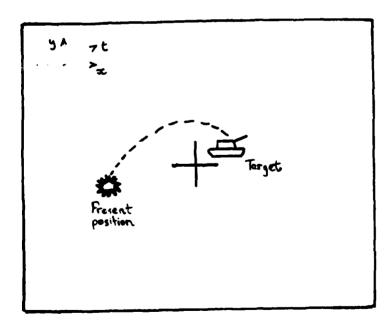
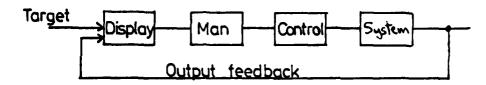
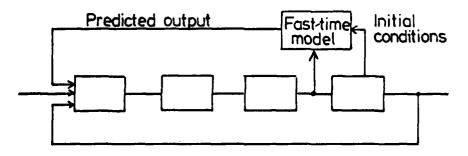


Figure 1b: Possible predictor display configuration for a missile guidance task.

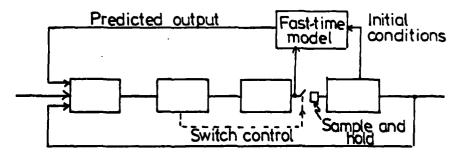
### (a) Unaided Control



### (b) On-line Prediction



### (c) Exploratory Prediction



### (d) Supervisory Prediction

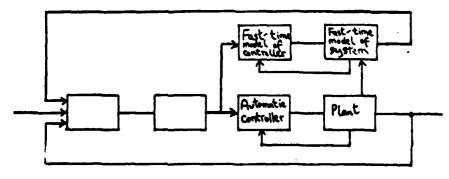
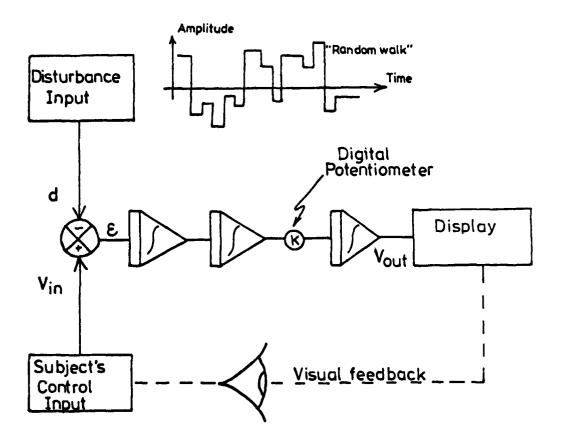


Figure 2: Possible modes of predictive control (after Warner, 1969).



$$V_{out} = \frac{K}{s^3} \cdot \epsilon$$

where  $\epsilon = (V_{in} - d)$ ,

 $S^3$  represents a third-order system,

K is the system gain.

Figures 3: Identical dynamics of the two simulated axes.

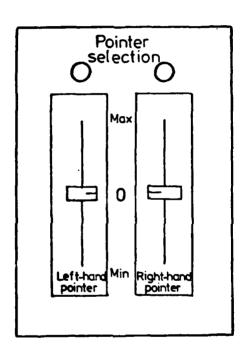


Figure 4: Subject's control panel

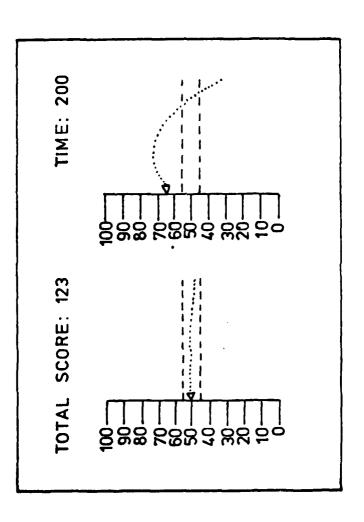


Figure 5: The display part-way through a trial with 30 seconds prediction span. (The predictor traces indicate the left-hand pointer to be relatively stable, wheras the right-hand pointer will pass through the limits and continue downwards. Note that only one of the pointers can be viewed at a given instance.)

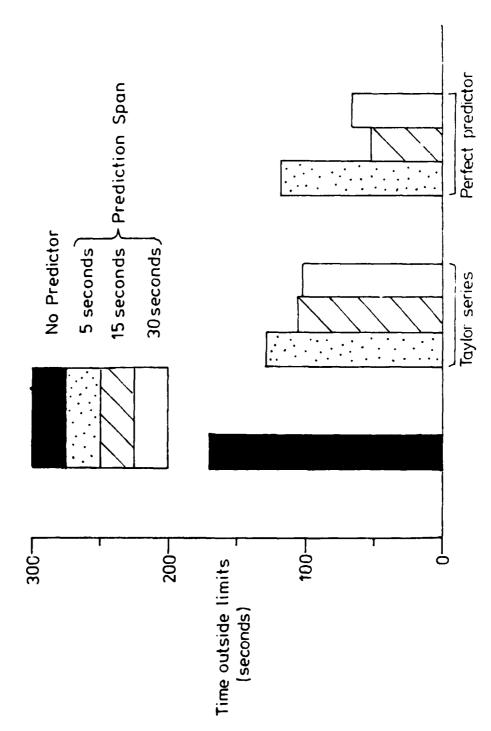


Figure 6: Grand averages of time outside limits scores

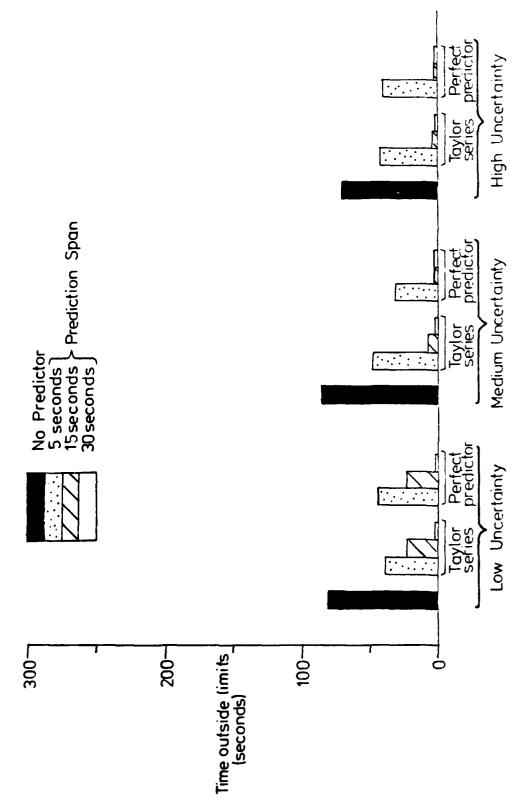


Figure 7: Time outside limits scores .. low gain (slow response time)

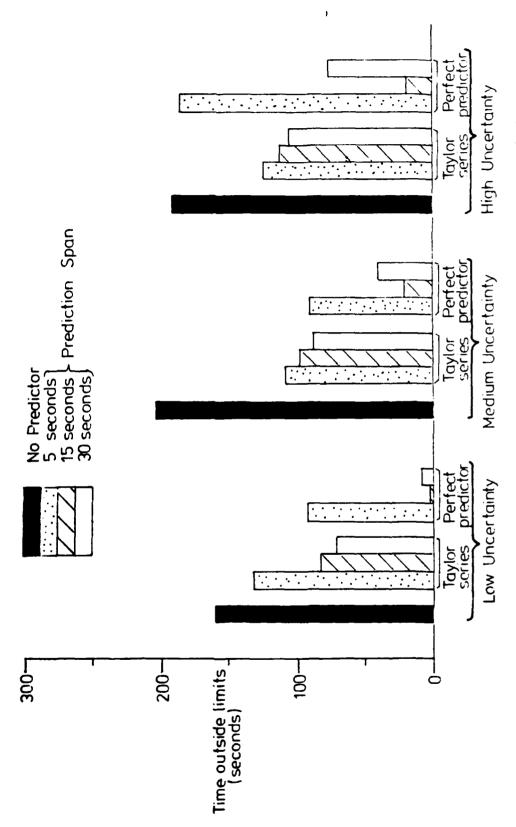


Figure 8 . Time outside limits scores .. medium gain (moderate response time)

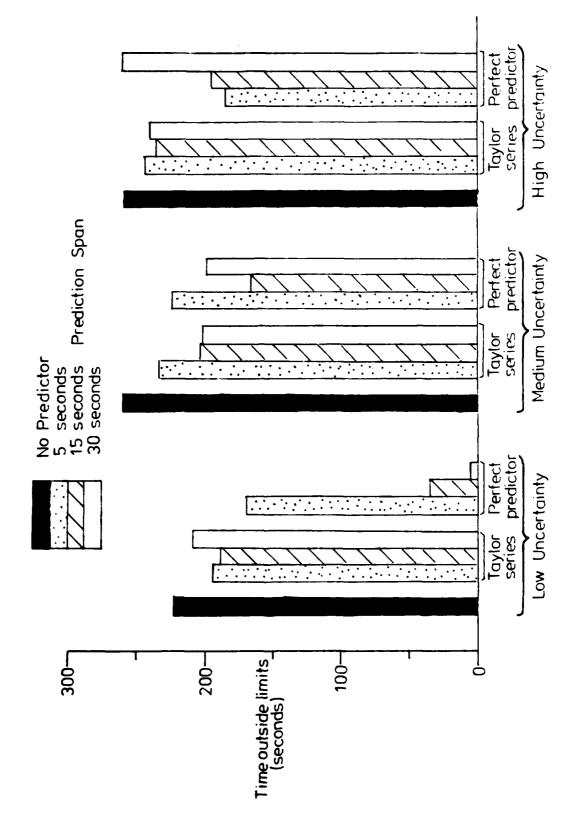


Figure 9: Time outside limits scores .. high gain (fast response time)

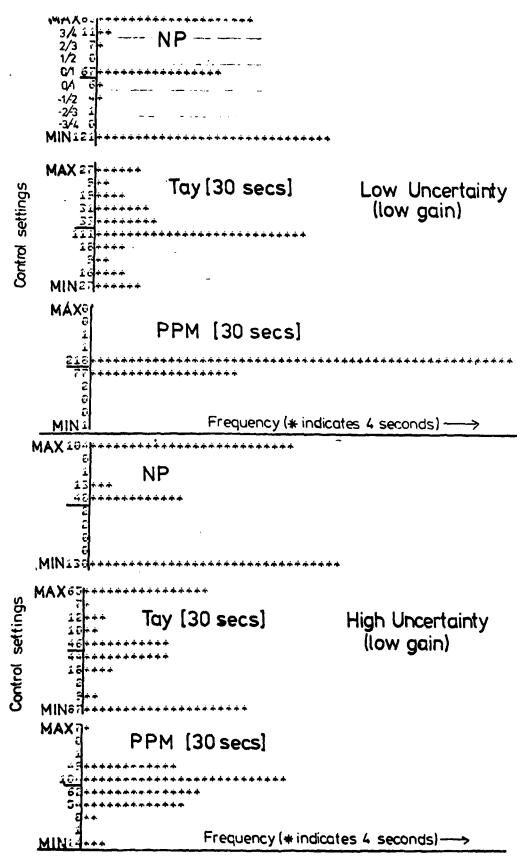


Figure 10. Control histograms - number of seconds during which control is within specified range settings.

TABLE 1 : SUMMARY ANOVA FOR LOG TRANSFORMED NP vs TAY (30 SECONDS)
vs PPM (30 SECONDS) SCORES (TIME OUTSIDE LIMITS DATA)

Source	Sum of Squares	df	Variance Estimate	'F'	Significance level
Between Subjects					
Gain	59.274	2	29.637	100.24	0.1% (df 2,12)
Subjects within groups	3.548	12	0.296		
Within Subjects					<del></del>
Uncertainty	4.757	2	2.379	25.82	0.1% (df 1,12)
Gain x Uncertainty	3.871	4	0.968	10.51	1% (df 2,12)
Uncertainty x S.w.g.	2.211	24	0.092		
Model (NP/Tay/PPM)	32.522	2	16.261	34.47	0.1% (df 1,12)
Gain x Model	9.657	4	2.414	5.12	5% (df 2,12)
Model x S.w.g.	11.321	24	0.472		
Uncertainty x Model	6.781	4	1.695	22.39	0.1% (df 1,12)
Gain x Uncertainty x Model	4.86	8	0.608	8.03	1% (df 2,12)
Uncertainty x Model x S.w.g.					

Conservative Test

TABLE 2 : SUMMARY ANOVA FOR LOG TRANSFORMED TAYLOR SERIES SCORES (TIME OUTSIDE LIMITS DATA.)

Source	Sum of Squares	df	Variance Estimate	'F'	Si	gnificance level	
Between Subjects							
Gain	82.827	2	41.413	32.98	0.1%	(df 2,12)	
Subjects within groups	15.07	12	1.256				
Within Subjects						<del>,</del>	
Uncertainty	0.0475	2	0.0238	0.16	_	(df 1,12)	
Gain x Uncertainty	0.7887	4	0.1972	1.29	-	(df 2,12)	
Prediction Span	13.66	3	4.553	13.05	1%	(df 1,12)	
Gain x Prediction Span	9.988	6	1.665	4.77	5%	(df 2,12)	
Prediction Span x S.w.g.	12.557	36	0.349				
Uncertainty x Prediction Span	0.455	6	0.0758	0.59	-	(df 1,12)	
Gain x Uncertainty x Prediction Span	2.065	12	0.1721	1.35	-	(df 2,12)	
Uncertainty x Prediction Span x S.w.g.	9.193	72	0.1277				

Conservative Test

### a) TESTS ON SIMPLE EFFECTS (Gain x Prediction Span Interaction)

Source	Significance
Between Gains at O seconds Prediction Span	5%
Between Gains at 5 seconds Prediction Span	0.17
Between Gains at 15 seconds Prediction Span	0.17
Between Gains at 30 seconds Prediction Span	0.1%
Between Prediction Spans at Low Gain	0.17
Between Prediction Spans at Medium Gain	5%
Between Prediction Spans at High Gain	-

TABLE 3 : SUMMARY ANOVA FOR LOG TRANSFORMED PERFECT PREDICTOR MODEL SCORES (TIME OUTSIDE LIMITS DATA)

Source	Sum of Squares	df	Variance Estimate	'F'	Significance level
Between Subjects					
Gain	48.448	2	24.224	45.76	0.1% (df 2,12)
Subjects within groups	6.353	12	0.529		
Within Subjects					
Uncertainty	7.866	2	3.933	13.88	1% (df 1,12)
Gain x Uncertainty	11.024	4	2.756	9.73	1% (df 2,12)
Prediction Span	49.706	3	16.569	37.92	0.1% (df 1,12)
Gain x Prediction Span	4.695	6	0.783	1.79	- (df 2,12)
Prediction Span x S.w.g.	15.73	36	0.437		
Uncertainty x Prediction Span	9.197	6	1.533	5.86	5% (df 1,12)
Gain x Uncertainty x Prediction Span	8.923	12	0.744	2.84	- (df 2,12)
Uncertainty x Prediction Span x S.w.g.	18.845	72	0.262		

Conservative Test

### a) TESTS ON SIMPLE EFFECTS (Gain x Uncertainty Interaction)

Source	Significance
Between Gains at Low Uncertainty	-
Between Gains at Medium Uncertainty	0.1%
Between Gains at High Uncertainty	0.1%
Between Uncertainty at Low Gain	-
Between Uncertainty at Medium Gain	17
Between Uncertainty at High Gain	0.17

### b) TESTS ON SIMPLE EFFECTS (Uncertainty x Prediction Span Interaction)

Source	Significance
Between Uncertainty at O seconds Prediction Span Between Uncertainty at 5 seconds Prediction Span Between Uncertainty at 15 seconds Prediction Span Between Uncertainty at 30 seconds Prediction Span	- 0.17 0.17
Between Prediction Spans at Low Uncertainty Between Prediction Spans at Medium Uncertainty Between Prediction Spans at High Uncertainty	0.1% 0.1% 0.1% (Just)

## END

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